What is a fraction?

BEAM

January 13, 2016

Problems:

What is a fraction?

$\frac{a}{b}$ represents the number $a \div b$.

The top of the fraction is the **numerator**. The bottom is the **denominator**. The denominator can never be 0, because you cannot divide by 0.

Two different fractions can be the same number. Since $6 \div 2 = 3$ and $12 \div 4 = 3$,

$$\frac{6}{2} = \frac{12}{4}.$$

These are actually *exactly the same thing*, just written differently, in the same way that "bunny" and "rabbit" are the same thing, just written differently.

Because $a \div a = 1$ for any a which is not 0, $\frac{a}{a} = 1$.

Two fractions are the same if you can multiply the numerator and denominator of one of them by the same number to get the other. For example:

$$\frac{2}{3} = \frac{8}{12} \qquad \text{because} \qquad \frac{2}{3} \times \frac{4}{4} = \frac{8}{12}.$$

$$\frac{\frac{2}{3}}{\frac{9}{3}} = \frac{2}{9} \qquad \text{because} \qquad \frac{\frac{2}{3}}{\frac{9}{3}} \times \frac{3}{3} = \frac{2}{9}.$$

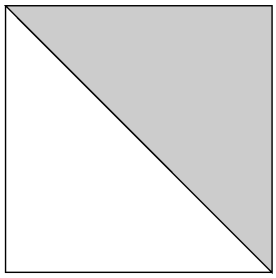
$$\frac{12}{8} = \frac{3}{2} \qquad \text{because} \qquad \frac{12}{8} \times \frac{\frac{1}{4}}{\frac{1}{4}} = \frac{3}{2}.$$

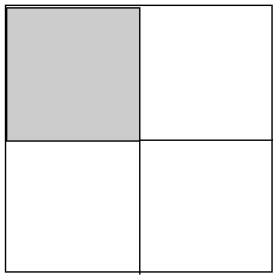
For any of these, we could have seen the same thing by multiplying in the other direction. For example, $\frac{3}{2} \times \frac{4}{4} = \frac{12}{8}$ and so $\frac{3}{2} = \frac{12}{8}$.

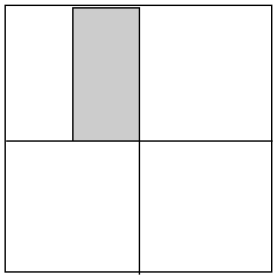
As you just saw, we can have fractions in the numerator or denominator. That's because a fraction is just a number, and any number can go into the numerator or denominator!

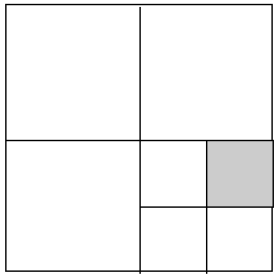
For any a, $\frac{a}{1} = a$ because $a \div 1 = a$.

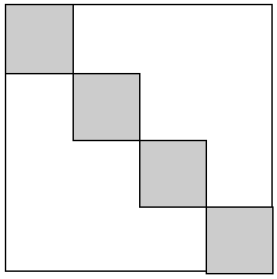
1. Write in fraction form the shaded part of each shape.

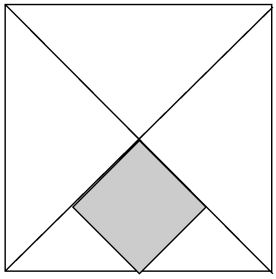
(a) 

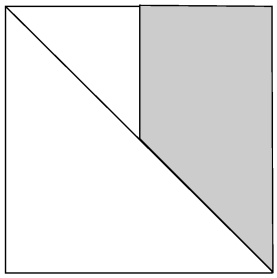
(b) 

(c) 

(d) 

(e) 

(f) 

(g) 

2. Problem 4.1.2 (page 155): Compute $\dfrac{16+6}{4-2}$.

3. Problem 4.1.4 (page 155): Which integer is closest to $\dfrac{43}{7}$?

4. At SPMPS 2012, there were 38 students. Twenty of them were girls and eighteen were boys. Jessica concludes that the fraction of the students that were girls was $\dfrac{38}{20} = \dfrac{19}{10}$. Was she right or wrong? If you think she was right, explain why. If you think she was wrong, give a different answer and explain why yours is correct.

5. What fraction of 96 is 64? Answer as a fraction in simplest form.

6. For each of the following, figure out which is larger in your head without making a common denominator. If you're not sure how to do that, ask a staff member.

   (a) $\dfrac{6}{3}$ vs. $\dfrac{8}{5}$.

   (b) $\dfrac{1}{10}$ vs. $\dfrac{1}{6}$.

   (c) $\dfrac{14}{24}$ vs. $\dfrac{17}{35}$.

(d) $\frac{4}{5}$ vs. $\frac{5}{6}$.

7. Fill in $<$, $>$, or $=$. Try to figure these out without putting them over a common denominator or "cross-multiplying" — use your intuition about fractions. Again, if you're not sure, ask one of us.

   (a) $\frac{1}{3}$ $\quad$ $\frac{1}{4}$

   (b) 4 $\quad$ $\frac{15}{4}$

   (c) $\frac{250}{350}$ $\quad$ $\frac{10}{14}$

   (d) $\frac{18}{5}$ $\quad$ $\frac{200}{111}$

8. Problem 4.1.7 (page 155): Find three fractions that are equal to $\frac{3}{5}$.

9. Place $\frac{4}{6}$ on the number line:

10. Find a fraction between $\frac{1}{9}$ and $\frac{1}{8}$.

11. Write $\frac{7}{12}$ as a fraction with denominator 132.