Rules of Exponents

BEAM

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Problems:

Rules of Exponents

Consider $5^{12} \cdot 5^7$. This expression tells you to multiply together 12 5's, and then to multiply in 7 more 5's. In total, that is 19 5's, so $5^{12} \cdot 5^7 = 5^{19}$. This is a general rule:

$$a^m \cdot a^n = a^{m+n}.$$

Now consider $(5^3)^{10}$. This tells you to multiply together 3 5's, and then to take the result and multiply it by itself 10 times. In the end, we get

$$\underbrace{5 \cdot 5 \cdot 5}_{5^3} \cdot \underbrace{5 \cdot 5 \cdot 5}_{5^3} \cdot \underbrace{5 \cdot 5 \cdot 5}_{5^3} \cdot \underbrace{5 \cdot 5 \cdot 5}_{5^3} \cdot \underbrace{5 \cdot 5 \cdot 5}_{5^3} \cdot \underbrace{5 \cdot 5 \cdot 5}_{5^3} \cdot \underbrace{5 \cdot 5 \cdot 5}_{5^3} \cdot \underbrace{5 \cdot 5 \cdot 5}_{5^3} \cdot \underbrace{5 \cdot 5 \cdot 5}_{5^3} \cdot \underbrace{5 \cdot 5 \cdot 5}_{5^3}.$$

That's 30 5's in total, so $(5^3)^{10} = 5^{30}$. This is also a general rule:

$$(a^m)^n = a^{m \cdot n},$$

because it is n a^m's which is $m \cdot n$ a's.

Finally, take a look at $3^5 \cdot 2^5$. If we look at the whole thing multiplied out, it's

$$\underbrace{3 \cdot 3 \cdot 3 \cdot 3 \cdot 3}_{3^5} \cdot \underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{2^5}.$$

We can move around the 2's and 3's to get

$$\underbrace{2 \cdot 3}_{6} \cdot \underbrace{2 \cdot 3}_{6} \cdot \underbrace{2 \cdot 3}_{6} \cdot \underbrace{2 \cdot 3}_{6} \cdot \underbrace{2 \cdot 3}_{6} = 6^5.$$

So $3^5 \cdot 2^5 = 6^5$. This is also a general rule:

$$a^n \cdot b^n = (ab)^n.$$

1. Express as a number raised to a single power:

   (a) $4 \cdot 4 \cdot 4 \cdot 4 \cdot 4$

(b) $18 \cdot 18^{25}$

(c) $2^4 \cdot 2^8$

(d) $5^7 \cdot 8^7$

(e) $2^{33} \cdot 2^{12} \div 2^8$

(f) $3^5 \cdot 9^5 \cdot 27^5$

(g) $(5^7)^3$

(h) $4^5 \cdot 7^{10}$

(i) $\left((14^3)^3\right)^3$

(j) $2^{34} \cdot 2^{20} \cdot (3^2)^{27}$

(k) $2^5 + 2^5$

(l) $4^5 + 4^5 + 4^5 + 4^5$

(m) $2 \cdot 19^{33} + 17 \cdot 19^{33}$

(n) $2^{100} - 2^{99}$

(o) $2^{100} - 2^{99} - 2^{98}$

(p) $2^{100} - 2^{99} - 2^{98} - \cdots - 2^{51} - 2^{50}$

2. Problem 2.2.2 (page 72): Compute the difference between the square of the cube of 2 and the cube of the square of 2.

3. What is $5^{23} \div 5^{20}$?

4. Explain why dividing by a^b is the same as multiplying by a^{-b}.

5. Problem 2.4.4 (page 88): Find the integer k such that $3^3 + 3^3 + 3^3 = 243 \cdot 3^k$. *(Source: MATHCOUNTS)*

6. Problem 2.4.5 (page 88): Express 2^{12} as a power of $\frac{1}{8}$.

7. Which is bigger and why: 2^{100} or $1,000,000$?

8. Problem 2.1.10 (page 63): The sum $1^2 + 2^2 + 3^2 + 4^2 + \cdots + 25^2$ is equal to 5525. Evaluate $2^2 + 4^2 + 6^2 + 8^2 + \cdots + 50^2$. *(Source: MOEMS)*

Exponents and Fractions

If you did the Simplifying Fractions module, then you have already done these!

9. Simplify:

(a) $\frac{3^2}{3}$

(b) $\frac{3^3}{3}$

(c) $\dfrac{3^3}{3^2}$

(d) $\dfrac{124^{30}}{124^{19}}$

(e) $\dfrac{50^9 \cdot 7}{50^8}$

(f) $\dfrac{3^2 \cdot 5^8 \cdot 11^{14}}{3^4 \cdot 5^6 \cdot 11^{13} \cdot 13}$

10. Problem 4.5.3 (page 177): Simplify the following fractions, assuming $a, b, m,$ and p are nonzero.

    (a) $\dfrac{4a^3b}{2ab}$

    (b) $\dfrac{8m^7p^{12}}{12m^5p^{15}}$

11. Problem 4.5.5 (page 177): Evaluate $\dfrac{42x^3y^6}{35x^2y^6}$ when $x = \dfrac{5}{4}$ and $y = \dfrac{2012}{2013}$.