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# BEAM Solvers

Congratulations to the successful solvers from Challenge Set 3!

**Top Solvers (4+ Solutions, win a prize!)**

- Aamirah Spencer
- Alex Torres
- BJ Kemp
- Brent Anderson
- Cindy Truong
- Daniel Myers
- Eils Thornton
- Gittan Johnson
- Hawa Drame
- Jacob Tom-Wong
- Jason Lin
- Justin Perez
- Liberty Sumners
- Maryam Yaqoob
- Mohammad Alhusaini
- Rawin Hidalgo

**Solvers (2+ Solutions)**

- Brianna Graham
- Camia King Cupid
- Dionna Perry
- Kali Sears
- Karen Zhao
- Melvin Chino-Hernandez
- Minhaj Uddin
- Yilin Li

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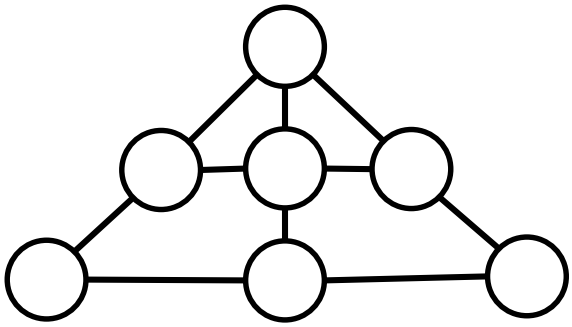
# Challenge Set 3 Solutions

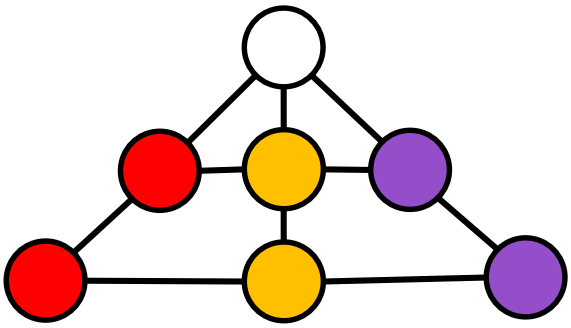
## Problem 1

Place each of the numbers 1, 2, 3, 4, 5, 6, and 7 in the circles at right so that any three circles connected by a line add to the same sum. For example, if the bottom three circles have numbers that add to 3, then each other line of three circles must also add to 3.



## Problem 1 Solution

Consider the red, orange, and purple pairs of circles highlighted here:



If you add the top circle (the white one) to any two circles of the same color, you get a line of three circles. So we must find numbers so that

the red circles + the white circle = the orange circles + the white circle = the purple circles + the white circle.

Because the white circle gets added to each of these sums, we just need the numbers in the colored circles to be equal:

the red circles = the orange circles = the purple circles.

Now look carefully at the numbers given:

1, 2, 3, 4, 5, 6, 7

Say that you put a 7 in the red circles. Because 7 is the biggest number, it should go with the smallest. In general, if we want to create pairs that have the same sum, it makes sense to try to pair the largest and smallest numbers together. So we could put 1 and 7 in the red, 2 and 6 in the orange, and 3 and 5 in the purple. This leaves only 4 to be put on the top circle.

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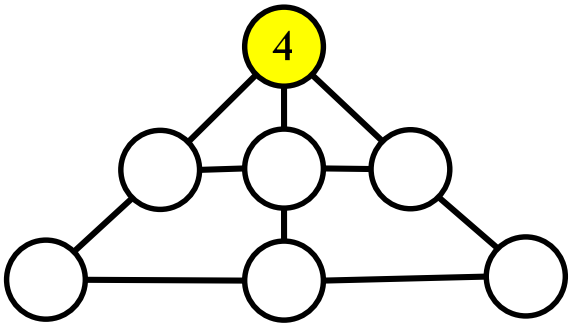
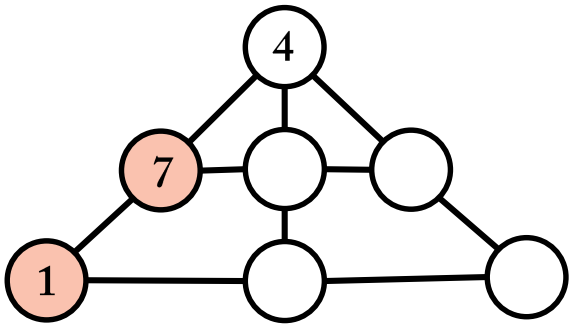
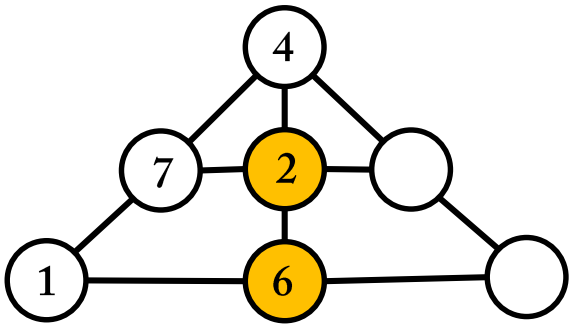
So we try with the 4 in the top circle:



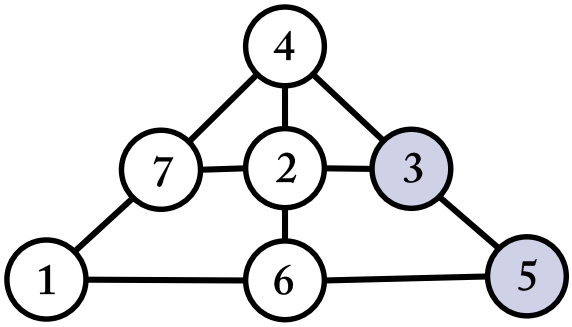

We can continue by placing 1 and 7 in the red circles – it doesn't matter which one is on top and which on bottom:




Now we have one entire line of three circles filled in! $4 + 7 + 1 = 12$, so if we did this part right, the other lines of three circles will also have to add to 12. We want to place 2 and 6 in the circles that were orange, but if we place 6 in the same line as 7, that will be too big since 6+7=13. So 6 has to be on bottom, like this:




Since $7 + 2 + 3 = 12$ and $1 + 6 + 5 = 12$, it is clear that we must place the last numbers like this:




At this point we should check that we didn't make any mistake and that all lines really do add to 12. They do, so we're done!

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## Problem 2

Lennin is running a board game activity! At the beginning of activities, every board game in the room is being played and there are three players in each game. Lennin doesn't play because he is helping everyone. At the end of activities, Lennin is also playing. This time, there are four players in each game, except one game that no one is playing. How many board games are there in the room?

## Problem 2 Solution

This problem can actually be solved with a careful guess-and-check. Because at the end, there are four players in each game, the number of players in total (including Lennin), must be a multiple of 4. It's also true that the number of players (without Lennin) has to be divisible by 3, since there are 3 players per game at the beginning of activities.

Could there be 4 players (including Lennin)? Then at the beginning of activities, there are 3 players (not including Lenin), and there must only be one board game. But at the end of activities, there's supposed to be a game no one is playing, which can't happen if there's only one board game. So there can't be 4 players total.

Could there be 8 players (including Lennin)? Then at the beginning of activities, there are 7 players (not including Lenin). But we can't divide up 7 players between board games so that each game has 3 players (there's always some players remaining!).

The same problem happens if there are 12 players (including Lennin): then at the beginning of activities, there are 11 players (not including Lennin), and we can't divide 11 players between games so that there are 3 players per game.

So could there be 16 players (including Lennin)? Then there would be 15 players (without Lennin) at the beginning of activities, so they could play 5 board games with 3 players each. At the end of activities, when Lennin joins, they then have 16 players, which means they could have 4 players playing 4 board games, meaning one board game is left over. This works! So the answer is there are 5 board games total in the room.

If you have learned about using variables to solve problems, you could use algebra to solve the problem, too! Use the variable $b$ to represent the number of board games. Then at the beginning of activities, we know there are 3 players for every board game, plus also Lennin, so the number of people in the room is:

$$3b + 1$$

At the end of activities, only b-1 board games are being played, but there are 4 players on each one, so the people in the room is:

$$4 \times (b - 1) = 4b - 4$$

The number of people in the room doesn't change, so these two equations must be the same. So we can write:

$$3b + 1 = 4b - 4$$

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Add 4 to both sides to get:

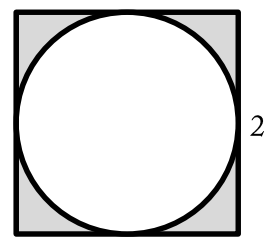
$$3b + 5 = 4b$$

Subtract $3b$ from both sides to get:

$$b = 5.$$

## Problem 3

At right is a square with side length 2 that has a circle removed from it. What is the area of the shaded region? You can leave your answer in terms of π.



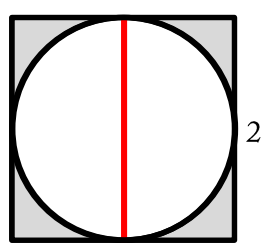

You may need to remember that the area of a circle is $\pi \times r^2$.

## Problem 3 Solution

The shaded region is the part that you get from taking the circle out of the square. Then the area of the shaded region is the area of the square minus the area of the circle.

The area of the square is the side length times itself, so it's $2 \times 2 = 4$.

To figure out the area of the circle, we first need to know the radius of the circle. Consider the red line in this figure:




The red line splits the square in equal size rectangles, so it has the same length as the side of the square, which is 2. The red line is also a diameter of the circle, which means it is twice as long as the radius. So the radius of the circle has length 1.

Now we can plug that into the equation given in the problem. The area of the circle is:

$$\pi \times r^2 = \pi \times 1^2 = \pi$$

So the area of the shaded region is $4 - \pi$.

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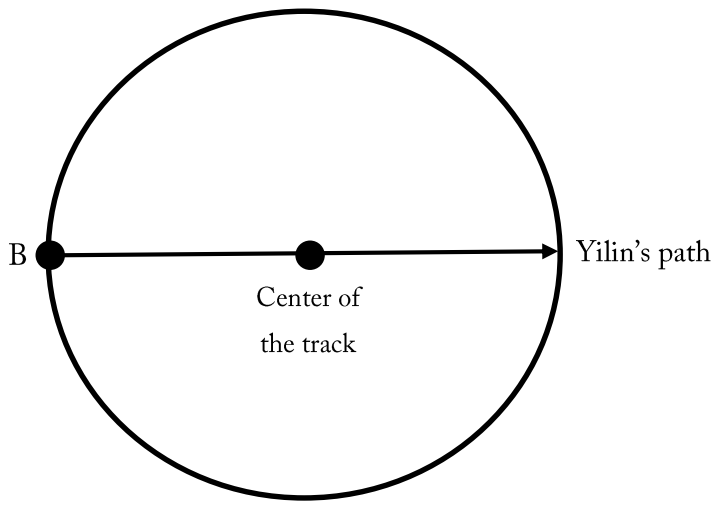
## Problem 4

Go to this website and watch the video

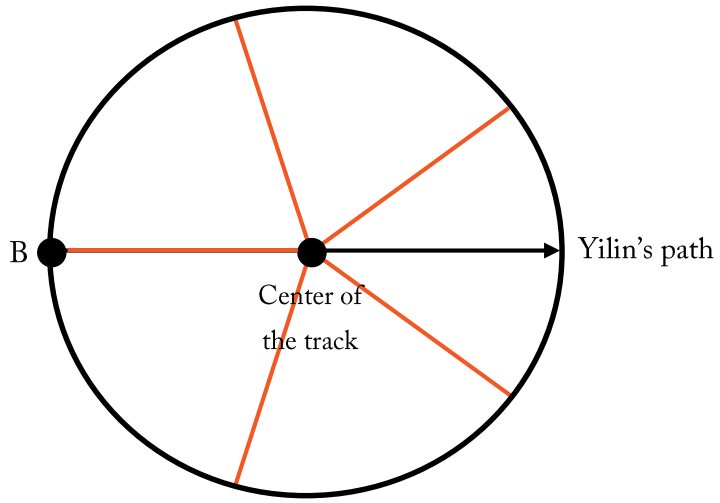
http://aops.com/videos/prealgebra/chapter10/197

Alex and Yilin are on a circular running track starting at the same place (point B). Alex starts running counterclockwise, and runs 4/5 of the way around the track. Yilin takes a shortcut and runs straight across the middle, as in the diagram below. From the center of the track, what is the angle between Alex and Yilin now?




## Problem 4 Solutions

We want to see where Alex will be on the circle. We start by dividing the circle into five pieces:



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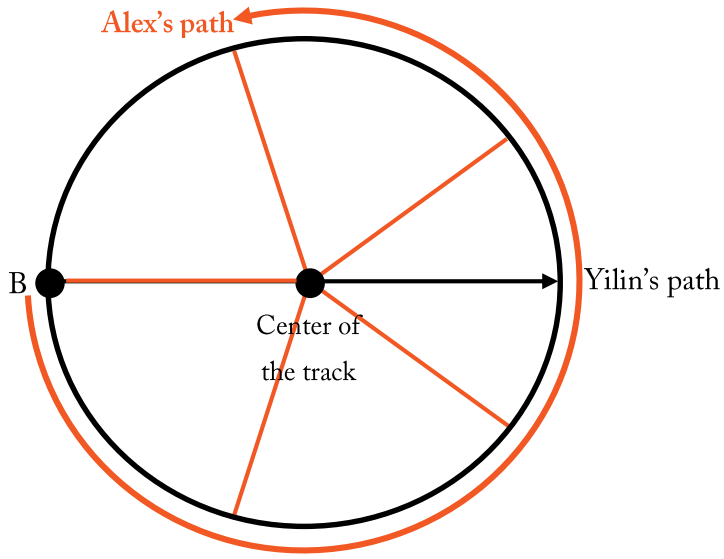
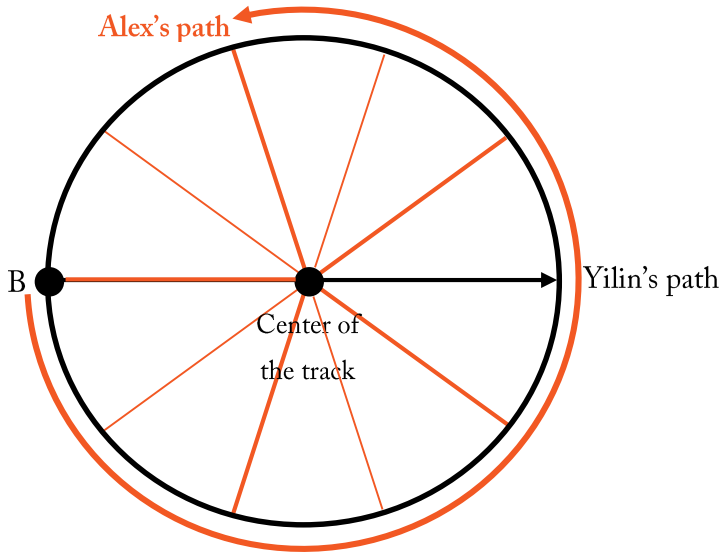
Since the entire circle is 360°, 1/5 of the circle in degrees is

$$360° \times \frac{1}{5} = \frac{360°}{5} = 72°.$$

Alex ran 4/5 of the way around counterclockwise, so he'll be at the 4th piece going around counterclockwise:




Yilin's path divides one of the 1/5 pieces in half, so we divide all of the 1/5 pieces in half to make them easier to count:




Half of a 1/5 piece is a 1/10 piece, so these are now 1/10 pieces. Each 1/10 piece is 36° since

$$\frac{72°}{2} = 360° \times \frac{1}{10} = 36°.$$

Now we count that Alex and Yilin are three 1/10 pieces apart. Since we showed that 1/10th of the circle is 36°, the answer is $3 \times 36° = 108°$.

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## Problem 5

On Ruthi's birthday, BEAM 6 students decide to pull a prank on her: some students will always lie to her, while other students will always tell the truth. Ruthi walks up to Jacob and Raynelis, and asks Jacob: "Does at least one of the two of you always tell the truth?" Jacob says either "yes" or "no" in response to this but you don't hear which. Based on Jacob's answer, Ruthi says she now knows whether Jacob always lies or always tells the truth and whether Raynelis does. Does Raynelis always lie or always tell the truth? You have to explain why your answer is correct and why no other answers could be correct.

## Problem 5 Solution

Let's call a student a liar if they always lie and call a student a truthteller if they always tell the truth. There are only four possibilities for who is a liar and who is a truthteller. Jacob and Raynelis could both tell the truth; they could both lie; Jacob could be a truthteller and Raynelis could be a liar; or Jacob could be a liar and Raynelis could be a truthteller. This is summarized in this chart:

| | **Jacob is a truthteller** | **Jacob is a liar** |
|---|---|---|
| **Raynelis is a truthteller** | Jacob: T, Raynelis: T | Jacob: L, Raynelis: T |
| **Raynelis is a liar** | Jacob: T, Raynelis: L | Jacob: L, Raynelis: L |

Let's write down all the possibilities and consider what Jacob would respond to Ruthi's question in each case.

> **Case 1:** Jacob is a liar and Raynelis is a liar. In this case, the true answer to Ruthi's question is no, but Jacob always lies, so he would answer **yes**.
>
> **Case 2:** Jacob is a liar and Raynelis is a truthteller. In this case, the true answer to Ruthi's question is yes, but Jacob always lies, so he would answer **no**.
>
> **Case 3:** Jacob is a truthteller and Raynelis is a truthteller. In this case, the true answer to Ruthi's question is yes, and since Jacob always tells the truth, he would answer **yes**.
>
> **Case 4:** Jacob is a truthteller and Raynelis is a liar. In this case, the answer to Ruthi's question is yes, and since Jacob always tells the truth, he would answer **yes**.

The problem says that as soon as Jacob answers the question, Ruthi knows who lies and who tells the truth! If Jacob said yes, then Case 1, Case 3, and Case 4 are all possible and Ruthi has no way of knowing which one it is. But if Jacob said no, then Ruthi knows that Case 2 is true and that Jacob is a liar and Raynelis is a truthteller. So Ruthi will only have enough information to know everything in Case 2, and so we know that we are in Case 2. Thus, Raynelis is a truthteller.

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## Problem 6

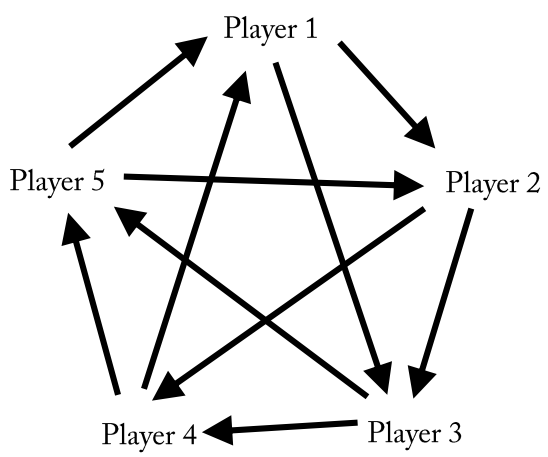
Five BEAM students decide to test themselves in a chess tournament. Each of them plays each of the others exactly once. There's no ties, so every game ends with a winner. At the end of the tournament, some of the students tie for first place (most number of wins). What is the largest number of first place winners possible? Give one example of how that can happen.

## Problem 6 Solution

There can be a 5-person tie. For example, one way this could happen:

- Player 1 wins against Player 2 and Player 3.
- Player 2 wins against Player 3 and Player 4.
- Player 3 wins against Player 4 and Player 5.
- Player 4 wins against Player 5 and Player 1.
- Player 5 wins against Player 1 and Player 2.

Sometimes it's helpful to think about problems like this by writing down the players and then drawing an arrow from the winner to the loser. The picture for the tournament described above looks like the figure below.



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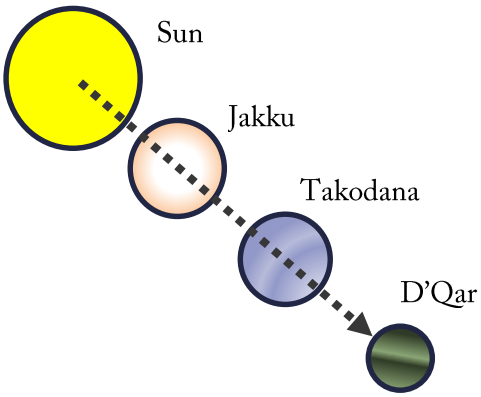
## Problem 7




The planets Jakku, Takodana, and D'Qar all orbit the same sun. Jakku takes 250 days to orbit all the way around to the same place, Takodana takes 300 days, and D'Qar takes 360 days. If the three planets are lined up in a line including the sun, as shown, what is the minimum positive number of days before they are all in the exact same locations again?

## Problem 7 Solution

If it takes Jakku 250 days to orbit all the way around to the same place, then it will be at the same position again only at multiples of 250, that is: 250, 500, 750, … This is also true about Takodana, but for multiples of 300, and D'Qar, but for multiples of 360. So the number we want is a multiple of 250, 300, and 360. Since we want the minimum number, this means we are looking for the least common multiple of 250, 300, 360.

Lots of people calculated the least common multiple by writing down lots of multiples of each of these numbers, but here's a trick: we can use primes! The least common multiple must be divisible by all the same primes as the numbers we started with, and no more. Begin by factoring:

$$250 = 2 \times 5 \times 5 \times 5$$

$$300 = 2 \times 2 \times 3 \times 5 \times 5$$

$$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$$

(If you know exponents, you can also write these as $2 \times 5^3$, $2^2 \times 3 \times 5^2$, and $2^3 \times 3^2 \times 5$.)

Any number that is a multiple of 250 must be divisible by 2 and by three 5's. Any number that is a multiple of 300 must be divisible by two 2's, a 3, and two 5's. Any number that is divisible by 360 must be divisible by three 2's, two 3's, and a 5.

If we find a number that is divisible by at least three 2's, two 3's, and three 5's, then that number will be a multiple of all three numbers at once. So to find the *least* common multiple, we multiply these numbers together and nothing else:

$$2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 5 = 9000.$$

(Hint: the quick way to do this is to see that $2 \times 5 = 10$, so you can simplify to $3 \times 3 \times 10 \times 10 \times 10$.)

The answer is 9000.