The Distributive Property

BEAM

January 13, 2016

Problems:

1. Ashlee tells Loquan that $5^2 + 12^2 = 169$. Loquan writes on his paper, $169 = 5^2 + 12^2$. Is what Loquan wrote correct? Why or why not?

2. For each of the following, mark the statement as true or false.

   (a) $2 + 3 = 5$
   (b) $3^2 - 4 = 5$
   (c) $2 + 3 = 3^2 - 4$
   (d) $169 = 13^2$
   (e) $5^2 + 12^2 = 13^2$
   (f) $(3^2 - 4)^2 + 12^2 = 13^2$

3. Henry tells Jayden that he's thinking of a number x where $(x - 8)^2 = 529$. Jayden tells Malachi that Henry is thinking of a number x where $(x-8)^2 - 29 = 500$. Is Jayden correct? Why or why not?

Suppose you're asked to compute $7(8 + 2)$. There are two ways to do this problem.

One way is to *distribute* first: to say that $7(8 + 2) = 7 \times 8 + 7 \times 2$. Then you can multiply and add: you get $56 + 14 = 70$. The other way is PEMDAS: to add what's in parentheses first, and then multiply. Then you get $7 \times 10 = 70$. There's no one right way to do this problem. Like anything in math, if each step you take is true, then everything you did is true.

I think the second way is easier, because you just have to do one small addition and one easy multiplication. But however you did it, it still works.

Have you ever stopped to ask why $7(8 + 2) = 7 \times 8 + 7 \times 2$?

Here's one way to think about it. 7×10 is "ten 7's", because that's what multiplication is: it's adding $\underbrace{7 + 7 + 7 + \cdots + 7}_{\text{10 times}}$. (That's why it's called "times"!) Since $10 = 8 + 2$, we can replace the 10 with $(8 + 2)$.

Now, ten 7's is the same as eight 7's plus two more 7's. Remember that we can replace 10 with $(8 + 2)$ because they're the same. So $7 \times (8 + 2) = 7 \times 8 + 7 \times 2$. It's like this:

$$\underbrace{7 \times 10}_{\text{ten 7's}} = \underbrace{7(8 + 2)}_{\text{still ten 7's}} = \underbrace{7 \times 8 + 7 \times 2}_{\text{eight 7's plus two more 7's}}$$

The **distributive property** tells you that this always works. It says:

For any three numbers a, b, and c,

$$a(b+c) = ab + ac.$$

For example, if $a = 7$, $b = 8$, and $c = 2$, then this says that $7 \times 10 = 7(8+2)$.

4. Ashlee gives Loquan the problem $7\times8+7\times2$. Loquan writes on his paper that $7\times8+7\times2 = 7(8+2)$ because of the distributive property. Then he adds $8+2$ to get 10, and says $7 \times 10 = 70$ which is the answer. Ashlee says that Loquan can't use the distributive property that way: you use the distributive property to go from $7(8+2)$ to 7×10, but you can't go the other way. She says that Loquan got lucky on this problem but that it won't always work. Who do you think is right? Why?

5. For each of the following, mark the statement as true or false.

   (a) $5 \cdot 30 + 12 \cdot 5 = 5(30+12)$

   (b) $12 \cdot 15 + 16 \cdot 2 = 12(15 + 16 \cdot 2)$

   (c) $5(2+3) = 5 \cdot 2 + 3$

   (d) $14(12+8) = 14 \cdot 12 + 14 \cdot 8$

   (e) $(5+3)4 = 5 \cdot 4 + 3 \cdot 4$

   (f) $18 - (6+4) = 18 - 6 + 4$

   (g) $6 \cdot 5 + 5 \cdot 3 = 3(2 \cdot 5 + 5)$

   (h) $127 + 127 + 127 + 127 + 127 = (1+1+1+1+1)127$

   (i) $2 \cdot 8 + 3 \cdot 9 + 4 \cdot 10 = (2+3+4)(8+9+10)$

   (j) $-5(-4+3) = 20 - 15$

6. Problem 1.3.6 (pg. 17): Using the distributive property, evaluate the following expressions.

   (a) $11 \cdot 43 + 11 \cdot 57$

   (b) $22 \cdot 6 + 6 \cdot 38$

   (c) $32 \cdot 16 + 16 \cdot 48$

7. Problem 1.3.8 (pg. 17): Compute $456+456+456+456+456+456+456+456+456+456$.

8. What is $(50 + 85 + 681 \cdot 5) \div 5$?

9. Problem 1.61 (pg. 51): Sean adds up all the even integers from 2 to 500, inclusive. Julie adds up all the integers from 1 to 250, inclusive. What is Sean's sum divided by Julie's sum? *(Source: MATHCOUNTS)*

10. Problem 1.74 (pg. 52): Let $a, b,$ and c be numbers. Simplify the expression $\big(a-(b-c)\big) - \big((a-b)-c\big)$. *(Source: MATHCOUNTS)*